



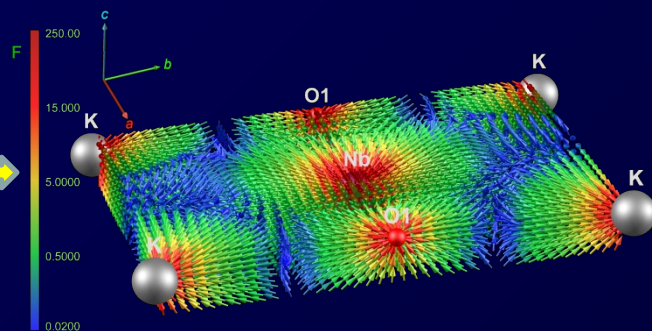
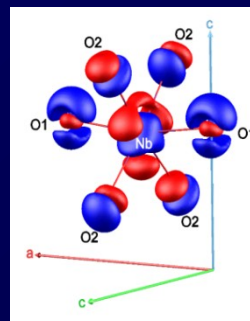
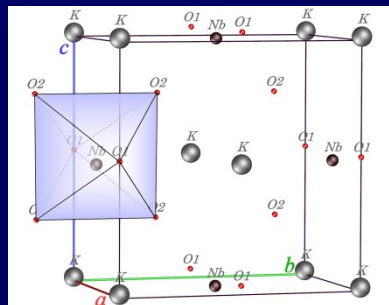
Российский химико-технологический университет им. Д.И. Менделеева

Кафедра квантовой химии

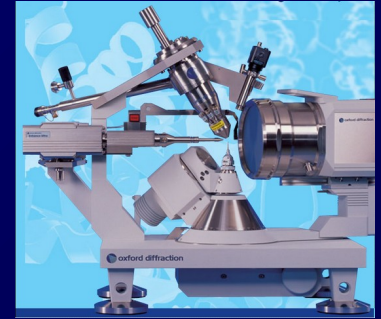
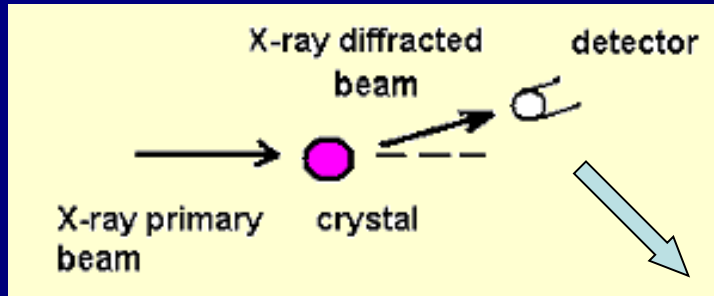


QUANTUM CRYSTALLOGRAPHY IS A TOOL OF THE FUTURE

By Vladimir G. Tsirelson



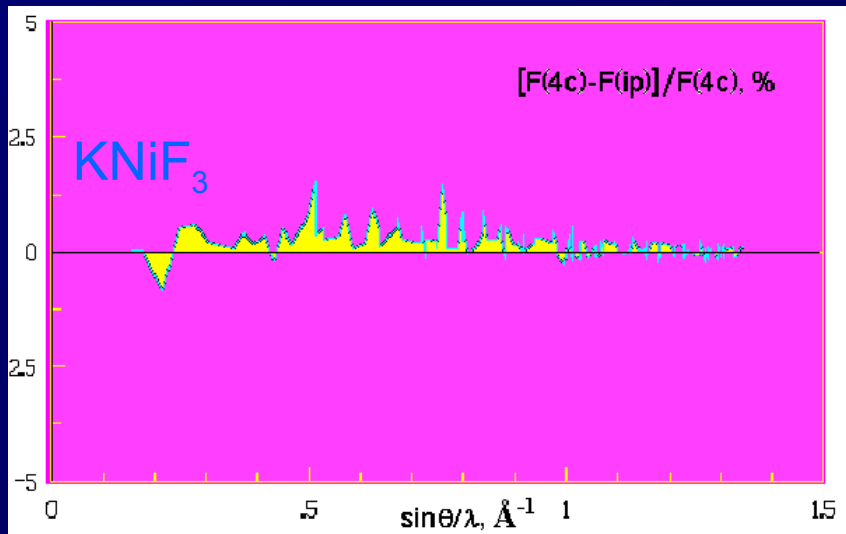
Derivation of the coherent X-ray structure factors



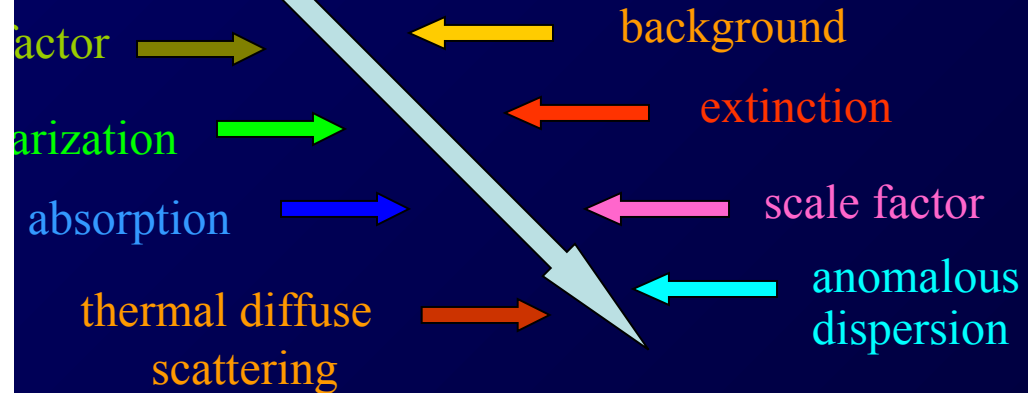
Born single-scattering

approximation

Total intensity



Relative difference between structure factors measured with point (4c) and area (ip) detectors



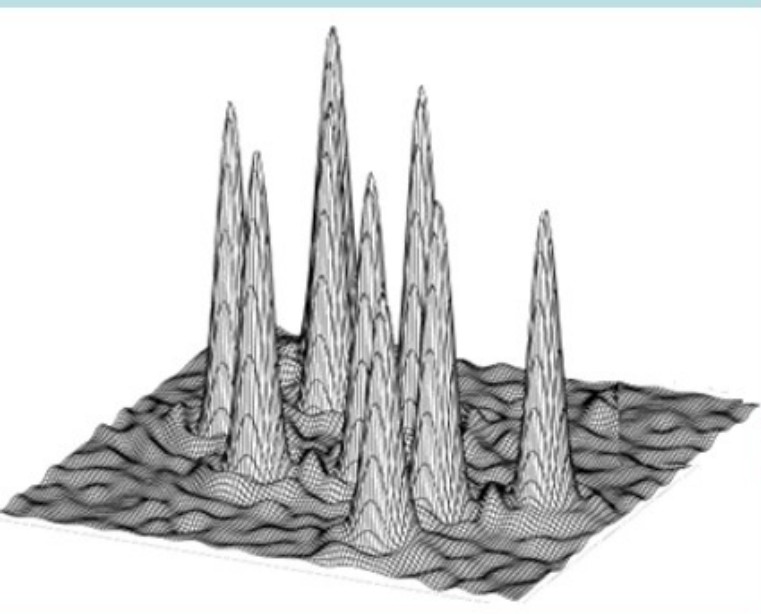
Elastic coherent

intensity $\sim F^2$

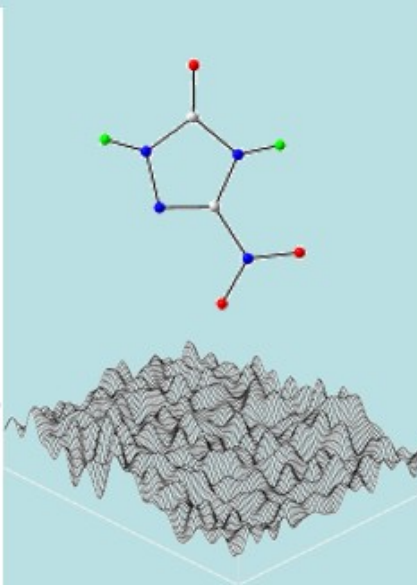
F^2

Reconstructing electron density

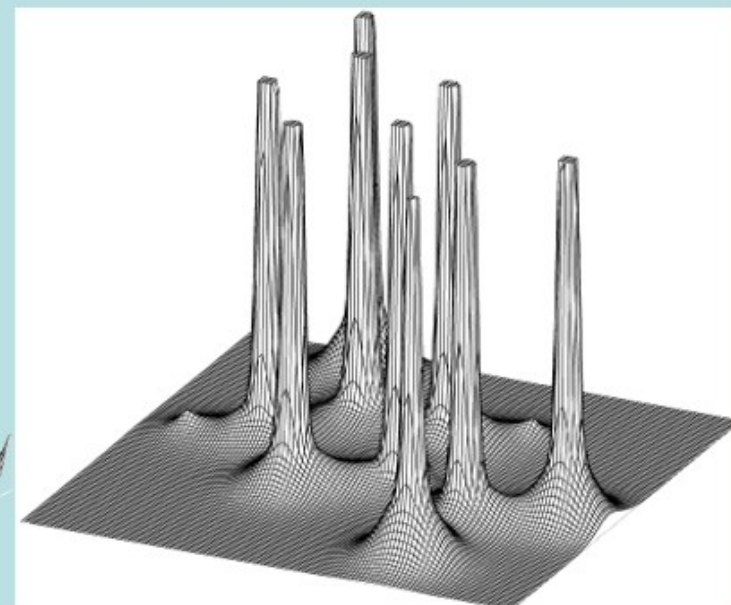
β -form of 5-nitro-2,4-dihydro-3H-1,2,4-triazol-3-one (NTO)



Dynamic electron density
from Fourier-series



*An experimental
noise*



Static electron density
from the multipole model

$$\rho(r) = \sum_i \rho_{\text{pseudoatom}} = \sum [C_m \rho_{\text{mono}} + C_d \rho_{\text{dip}} + C_q \rho_{\text{quadr}} + C_o \rho_{\text{octo}} + C_{\text{hexa}} \rho_{\text{octo}}]$$

The structural space-distributed multipole model

QTAIM – Quantum Theory of Atoms in Molecules and Crystals.
 Important notions of QTAIM are atoms and atomic interactions.

The key structural elements of QTAIMC, based on electron density, are:

a) the critical points:

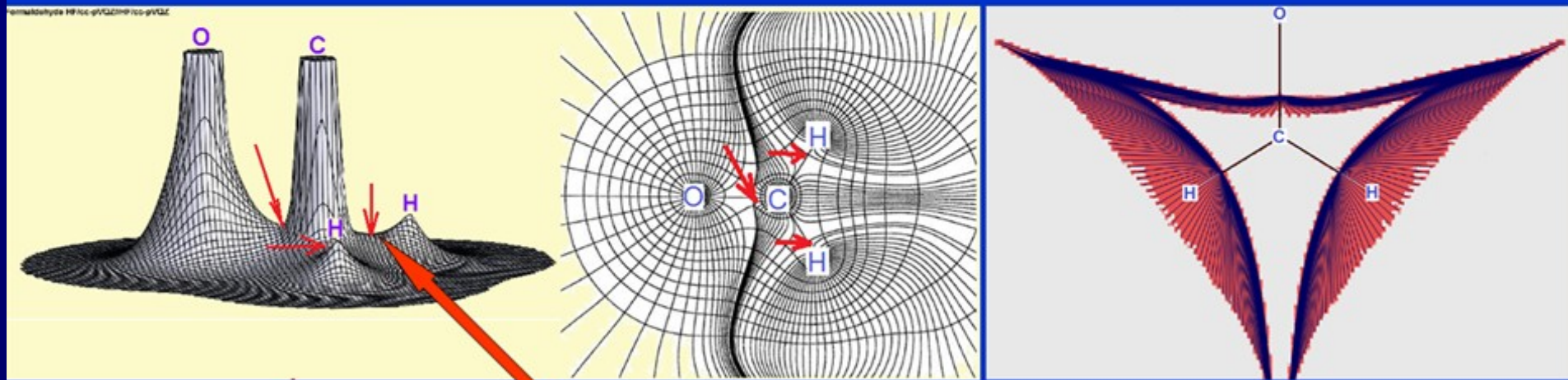
$$\nabla\rho(\mathbf{r}) = 0$$

Each CP is characterized by the three principal ED curvatures

The bond critical point is a saddle (3,-1) point between atoms

b) the zero-flux interatomic surfaces separate the atoms

$$\nabla\rho(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}) = 0$$



Electron density peaks on the nuclei

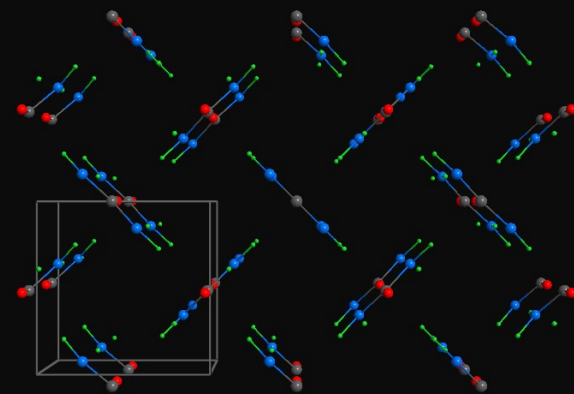
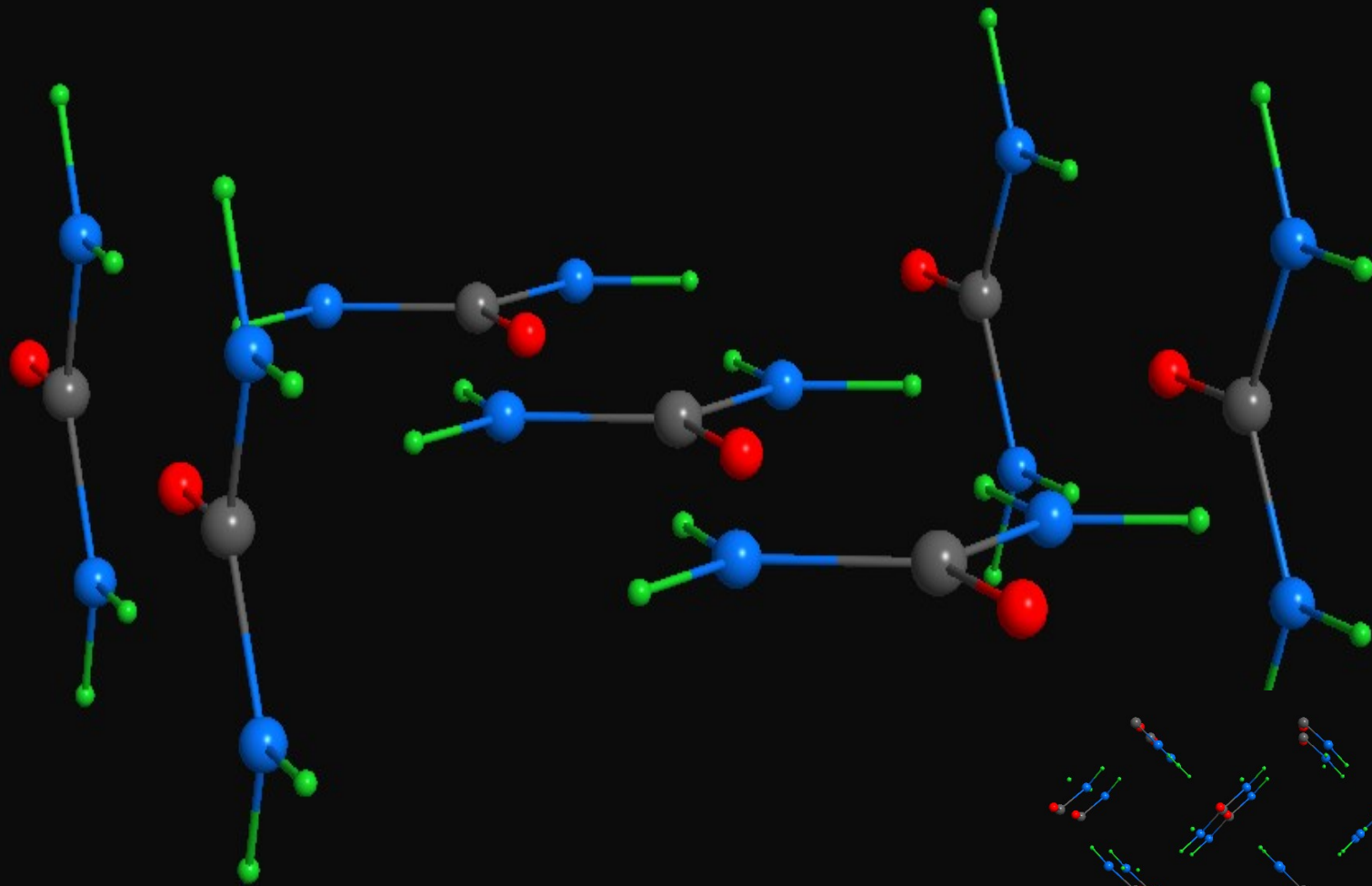
Separate bonds/interactions are not
 observables

The bond path is associated with bonded atomic interaction (a postulate)

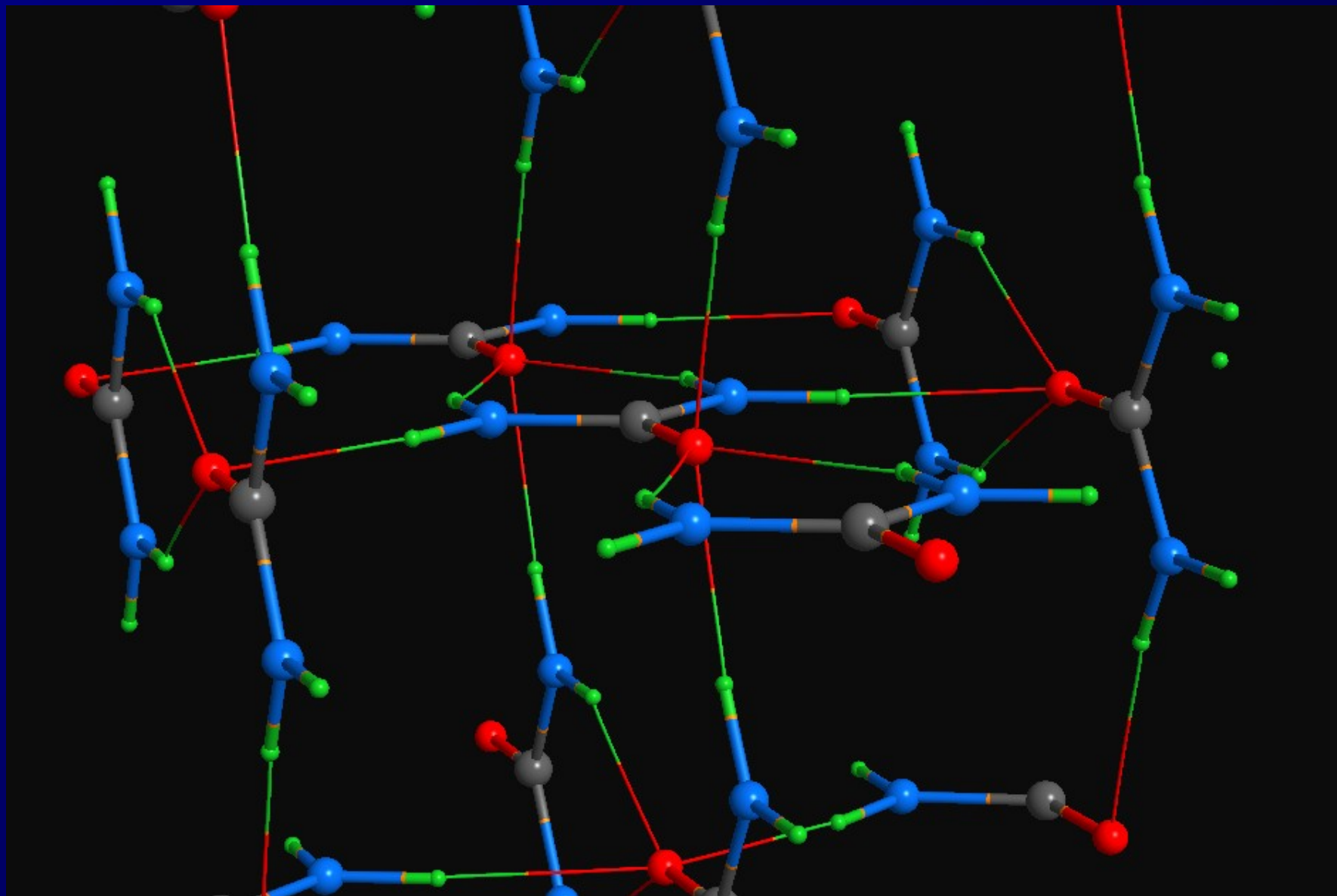
Bader and coworkers (1972-1981): principles of QTAIMC

Tsirelson and coworkers (1984-1988): application of QTAIMC to experimental electron densities

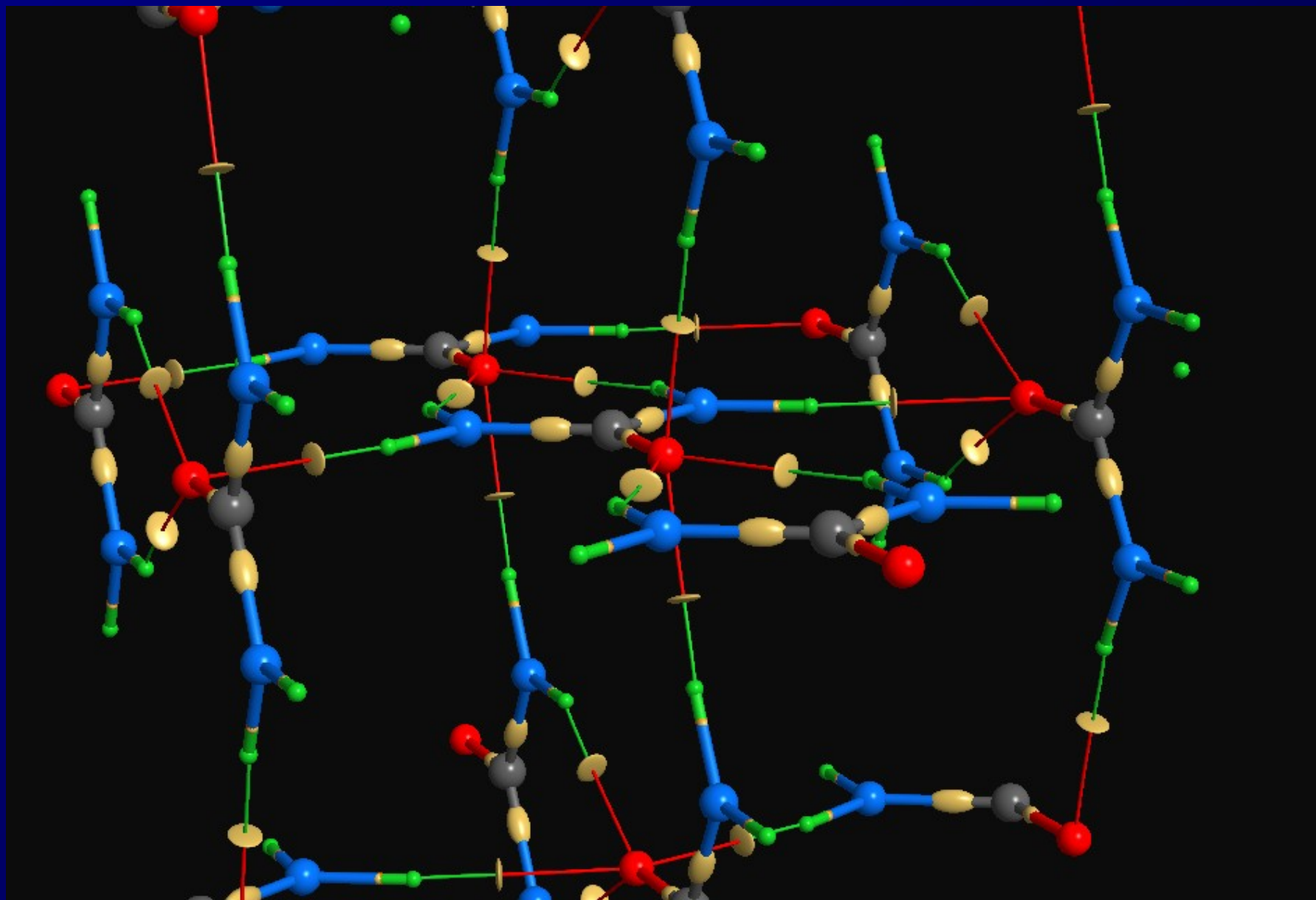
Urea $\text{CO}(\text{NH}_2)_2$: explicit presentation of the bonding features



Urea $\text{CO}(\text{NH}_2)_2$: explicit presentation of the bonding features



Urea $\text{CO}(\text{NH}_2)_2$: explicit presentation of the bonding features

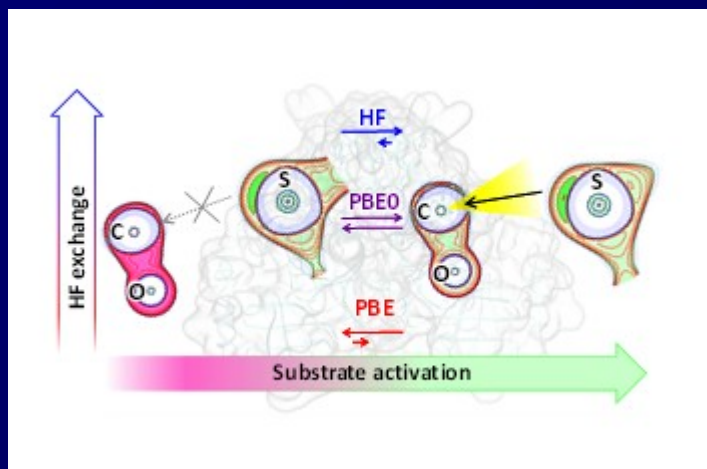


Молекулярный механизм субстратной специфичности основной протеазы SARS-CoV-2

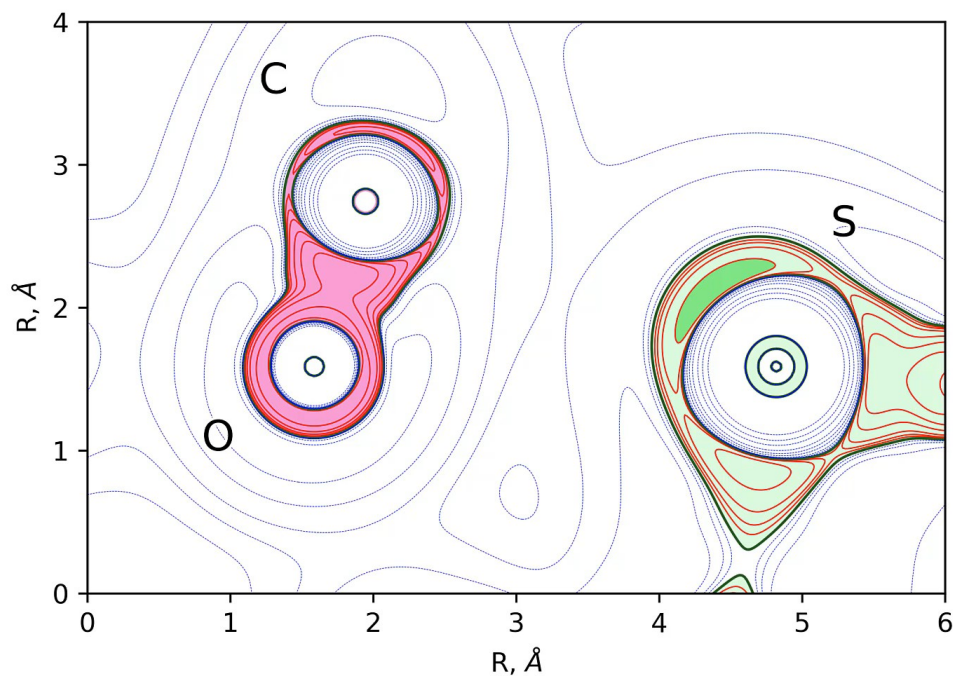
Методом молекулярной динамики с потенциалами КМ/ММ оценены электронные свойства активного центра фермента с различными субстратами вдоль динамических траекторий. Картины активации субстрата ферментом, позволили различить реакционные и нереакционные комплексы. Рассчитанные доли реакционных конформаций фермент-субстратных комплексов вдоль траекторий коррелируют с данными по субстратной специфичности основной протеазы SARS-CoV-2. Важно для создания селективных ингибиторов этого фермента.

PhysChemChem Phys

(Hot paper, 2020).



The Laplacian of electron density in the plane formed by S atom of Cys145, and C-O bond of the carbonyl group of substrate



Definition:

Quantum crystallography is a research area exploiting the fact that parameters of quantum-mechanically valid electronic model of a crystal can be derived from the accurately measured set of X-ray coherent diffraction intensities, $|F(\mathbf{q})|^2$

$F(\mathbf{q})$ are the Fourier transforms of the position-space dynamic one-electron density, which reflect the atomic structure and displacements, as well as the chemical bonding features.

Orbital-free quantum crystallography operates with the electron density and its derivatives and avoids the orbital approximation (Hohenberg-Kohn version of DFT)

V. Tsirelson & A. Stash. Orbital-Free Quantum Crystallography: View on Forces in Crystals. Acta Cryst. 2020, B76, 769.

V. Tsirelson & A. Stash. Developing Orbital-Free Quantum Crystallography: The Local-Potentials and Associated Partial Charge Densities. Acta Cryst. 2021, B77, 467.

Hohenberg & Kohn (1964):

1) Electron density uniquely defines the ground electronic state of atoms, molecules and crystals

2) Electron density provides a minimum of the energy

functional

$$E[\hat{\psi}] = G[\hat{\psi}] + E_{ne}[\rho] + E_{ee}[\hat{\psi}] + E_{xc}$$

kinetic energy $[\rho]$ *exchange & correlation energy*

electron-nuclear energy *electron-electron energy*

Electrostatics

Forces in molecules and crystals:

- The Hellmann-Feynman (classic-like) force acts on the nuclei;
 - The Ehrenfest (classic + quantum) force acts on the electrons;
 - The quantum force linked with electron motion: acts on the electrons.
- ↑
Pauli principle +
electron correlation

All the local resulting forces in a system at equilibrium are zero

The Kohn–Sham one-electron DFT potential acting on noninteracting electrons yields the ground-state density of a real system. The Euler equation for noninteracting electrons in a stationary ground state

$$\mu[\rho] = \frac{\delta T_s[\rho]}{\delta \rho} + v_{eff}[\rho]$$

μ is chemical potential, which follows from the condition $\int \rho(\mathbf{r}) d\mathbf{r} = N$.

$\mu = \text{Const.}$

The orbital-free DFT is based on the density approximations to the Kohn-Sham equations.

Static potential

$$v_{eff}(\mathbf{r}) = -\sum_A \frac{Z_A}{|\mathbf{r}-\mathbf{R}_A|} + \frac{1}{\rho(\mathbf{r})} \int \frac{\pi(\mathbf{r},\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

$\pi(\mathbf{r},\mathbf{r}')$ is the pair electron density.



A single-determinant DFT analog of $v_{eff}(\mathbf{r})$ is the static potential acting on an electron in a molecule (PAEM):

$$v_{paem}(\mathbf{r}) = -v_{esp}(\mathbf{r}) + v_x(\mathbf{r}).$$

The orbital-free exchange potential may be approximated according to LDA, Becke, etc.

Kinetic potential

Single-determinant noninteracting kinetic energy, $T_s[\rho]$, is exactly presented as a sum of the Weizsäcker, $T_W[\rho]$, and Pauli, $T_P[\rho]$, contributions:

$$T_s[\rho] = T_W[\rho] + T_P[\rho].$$

Energy $T_W[\rho]$ originates from the wave-particle duality of electrons and uncertainty principle. It can be viewed as kinetic energy of noninteracting “spinless” particles of density $\rho(\mathbf{r})$, which are in the “bosonic” ground state (all particles are in the same lowest energy state).

The Pauli energy, $T_P[\rho]$, describes the excess in the total electronic kinetic energy over the energy of the noninteracting “spinless” particles. It follows from the demand of Fermi-Dirac statistics for the many-electron wavefunction.

Taking the functional derivative of $T_s[\rho]$ we get

$$v_{kin}(\mathbf{r}) = v_W(\mathbf{r}) + v_P(\mathbf{r})$$

$$v_W(\mathbf{r}) = \frac{1}{8} \frac{|\nabla\rho(\mathbf{r})|^2}{\rho^2(\mathbf{r})} - \frac{1}{4} \frac{\nabla^2\rho(\mathbf{r})}{\rho(\mathbf{r})}.$$

$$v_P(\mathbf{r}) \approx -v_{PAEM}(\mathbf{r}) - v_W(\mathbf{r}) + \mu$$

Bringing together all the local potentials, we arrive at the Euler equation in the form

$$\mu = v_{kin}(\mathbf{r}) + v_{PAEM}(\mathbf{r})$$

Static potential $v_{PAEM}(\mathbf{r}) = -v_{esp}(\mathbf{r}) + v_x(\mathbf{r})$ depends on the equilibrium nuclear configuration and on corresponding electron density.

Kinetic potential $v_{kin}(\mathbf{r}) = v_p(\mathbf{r}) + v_w(\mathbf{r})$ contains the terms related with electron motion.

The electron density acts as a main variable now.

The kinetic and static potentials and related force fields in a crystal

Chem. potential for stable system is constant $\rightarrow \nabla\mu(\mathbf{r}) = 0$
and the total (net) local force acting on electron in a
molecule/crystal at \mathbf{r} is:

$$\begin{aligned}\mathbf{F}(\mathbf{r}) &= -\nabla\mu(\mathbf{r}) \Rightarrow -\nabla v_{kin}(\mathbf{r}) - \nabla v_{PAEM}(\mathbf{r}) = \\ &= \mathbf{F}_{kin}(\mathbf{r}) + \mathcal{F}(\mathbf{r}) = 0\end{aligned}$$

$\mathcal{F}(\mathbf{r}) = \mathbf{F}_{PAEM}(\mathbf{r}) = \mathbf{F}_{esp}(\mathbf{r}) + \mathbf{F}_x(\mathbf{r})$ describes the interaction
of any one electron of molecule/crystal with the averaged
distribution of the remaining (N-1) electrons and all the nuclei.

$\mathbf{F}_{kin}(\mathbf{r}) = \mathbf{F}_P(\mathbf{r}) + \mathbf{F}_W(\mathbf{r})$ is the force per electron of
kinetic origin.

$\mathcal{F}(\mathbf{r})$ 

Common fundamental tendency to a leveling of distinctions is the trend towards to homogeneous disorganization or to the *homotropy* (Revici, 1961). The factors preserving homogeneous disorganization in a system express the *heterotropy* of the matter. These two opposing trends determine the structure and properties of substances at all levels of their spatial organization.

 $F_{kin}(\mathbf{r})$ 

Availability of many nuclear centers of attraction in crystals leads to a redistribution of electron density, i.e. to formation of the structurally and functionally stable objects. Due to *homotropic and heterotropic forces*, atomic electron shells are reorganized, depending on the confinement (i.e. structural) features. They form open atomic-like systems, the bonded atoms, which are in the dynamic mechanical equilibrium state, each of them has a certain shape, electron population, distribution of electron density and energy value. Resulting picture of forces within whole system is again balanced by the mutual influence of bonded atoms.



A Program for Calculating Crystal and Molecular Properties Using Multipole Parameters of the Electron Density

by

Adam Stash and Vladimir Tsirelson

J. Appl. Cryst. (2022). 55, 420–424

WinXPRO64 - [D:\Compounds\CAF2\17_07_2016\vcf2_...]

File Edit View Tools Help

Current wavefunctions bank - C:\Program Files (x86)\WinXPRO64\wavefunREL.bnk

* The active property is RHO

- current kinetic energy approximation: Kinetic gradient energy (Kirzhnits,1957)
- current exchange energy approximation: Becke (1988) (BECKE)
- current correlation energy approximation: Chachiyo-Karasev (ChK)
- current exchange potential approximation: von Barth, Hedin (BARTH-HEDIN)
- current correlation potential approximation: Chachiyo-Karasev (ChK)

Properties

Electron Density and its Derivations

- Total electron density [RHO]
- Core electron density [CORE]
- Valence electron density [VALENCE]
- Deformation density [DD]
- Laplacian of RHO [d2RHO]
- Gradient of RHO [dRho]
- E. s. d. in total density [SigRHO]
- E. s. d. in Laplacian of RHO [SigLAPL]

Local Potentials

- Nuclear electrostatic potential [NPOT]
- Electronic electrostatic potential [EPOT]
- Total electrostatic potential [ESPOT]
- One-electron potential [OEP]

Exchange potential [eX-POT]

von Barth, Hedin (BARTH-HEDIN)

Correlation potential [CorPOT]

Chachiyo-Karasev (ChK)

Exchange-Correlation potential [xcPOT]

- Pauli potential [PaulPOT]
- Quantum (fermionic) potential [QuantPOT]
- von Weizsacker (bosonic) potential [WeizPOT]

Potential acting on electron in molecule [PAEM]

- Kinetic potential [KinPOT]
- Take into account the correlation potential [Vc]

Local Charge Density

- Exchange charge density [ExChD]
- Correlation charge density [CorChD]
- Exch-Corr charge density [xcChD]
- Pauli charge density [PaulChD]
- Quantum (fermionic) charge density [QuantChD]
- von Weizsacker (bosonic) charge density [WeizChD]
- PAEM charge density [PAEMChD]
- Kinetic charge density [KinChD]

Local Energy Characteristics

Kinetic energy approximation

Kinetic gradient energy [Kirzhnits,1957]

- Kinetic energy density [KINET]
- Potential energy density [POTEN]
- Electronic energy density [ELECTRON]
- Schroedinger kinetic energy density [SHROED]
- Electron localization function [ELF]
- Localized-orbital locator [LOL]
- Localized-orbital locator 2 [LOL2]
- Local temperature <kT> [LTEMP]
- Local Wigner-Parrr entropy <s/k> [ENTROPY]
- Local binding entropy <sb/k> [BNDENTRP]
- Local Shannon entropy [SHENTROPY]
- von Weizsacker energy density [WeizEnD]
- Fisher information [FishINF]
- Phase-Space defined Fisher Inform. Density [PS-FID]

Exchange Fluid Pressure

- Non-interacting kinetic electron pressure [KEP]
- Exchange Pressure [ExP]
- Correlation pressure [LDACP]
- Total quantum pressure [KEP+ExP+LDACP] [PQUANT]
- Scale pressure (relative to Thomas-Fermi gas) [SPTF]
- Total electrostatic pressure density [P_ELST]
- Nuclear electrostatic pressure density [Pn_ELST]
- Electronic electrostatic pressure density [Pe_ELST]
- Total electron pressure [PTOTAL]
- Indicator of quantum pressure focusing [IQFF]

Exchange Energy Density [XED]

Becke (1988) (BECKE)

Exchange energy density per particle (XEDPP)

Correlation Energy Density [CED]

Chachiyo-Karasev (ChK)

Correlation energy density per particle (CEDPP)

State _____

Spin-compensated Ferromagnetic

Miscellaneous

- Fermi momentum [FERMI]
- Atomic Dipole Moment [ADM] (requires integration within atomic basin)
- Probability density function [PDF] (PDF will be calculated for an independent atom selected from the list)
- Reduced Density Gradient [RDG] RHO*sign(Lambda2)
- RHO*sign(Lambda2) [RHOsl2]

Fermi Hole and correlation densities

RHO(i) =

- Fermi hole [FHOLE]
- Pair correlation density [PAIRCD]
- Conditional pair correlation density [CPAIRD]
- Conditional pair correlation density [CPRDx2]

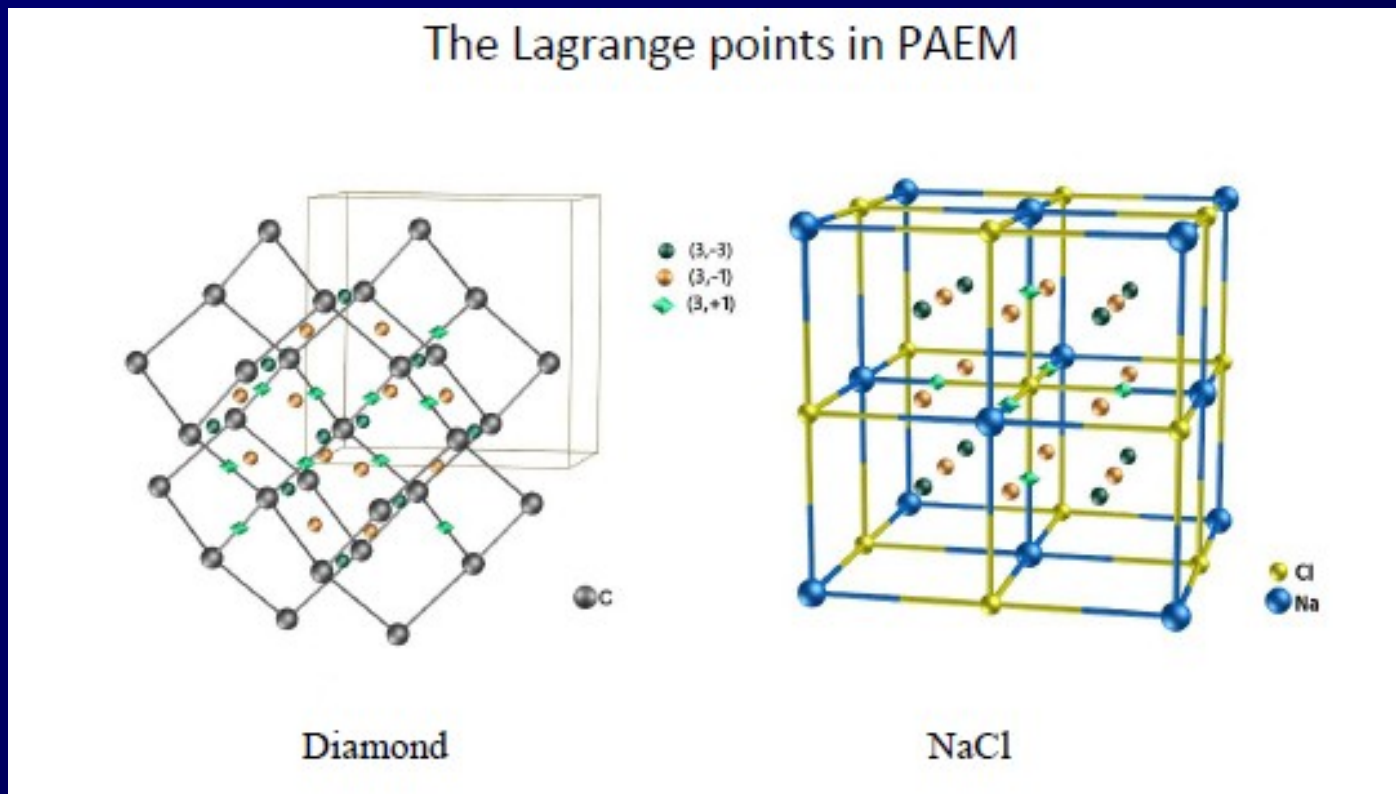
Common part

- * Cell based cluster
- Set negative sign of the current property
- Set Laplacian of RHO
- Cut off above Rho = e/A^3 a.u.
- Cut off lower Rho = e/A^3 a.u.

Current property is RHO

The Lagrange critical points in $kin(\mathbf{r})$ and the points at which both the net force and forces \mathbf{F} separately are zero.

These CPs are of **mechanical origin** and differ from the Bader's QTAIM critical points, which are of **topological origin**.



The critical points of two scalar fields, $v_{kin}(r)$ and $v_{PAEM}(r)$ at the same positions are mutually related as follow:

$$(3, +3)_{PAEM} \rightarrow (3, -3)_{kin};$$

$$(3, -3)_{PAEM} \rightarrow (3, +3)_{kin};$$

$$(3, -1)_{PAEM} \rightarrow (3, +1)_{kin};$$

$$(3, +1)_{PAEM} \rightarrow (3, -1)_{kin}.$$

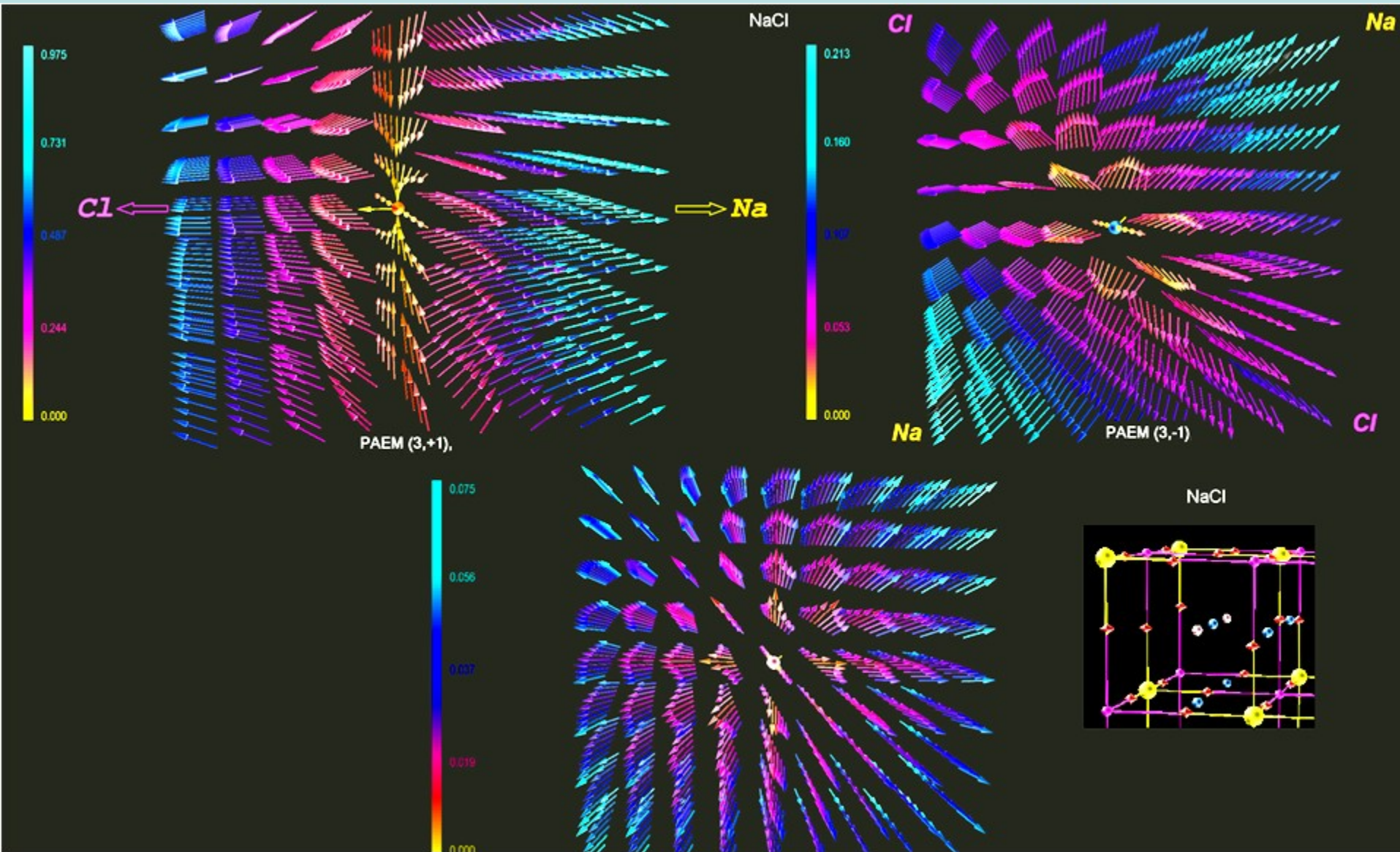
The fields $v_{PAEM}(r)$ and $v_{kin}(r)$ exhibit different location of the three-dimensional attractors

Diamond

Link to the classical crystallography

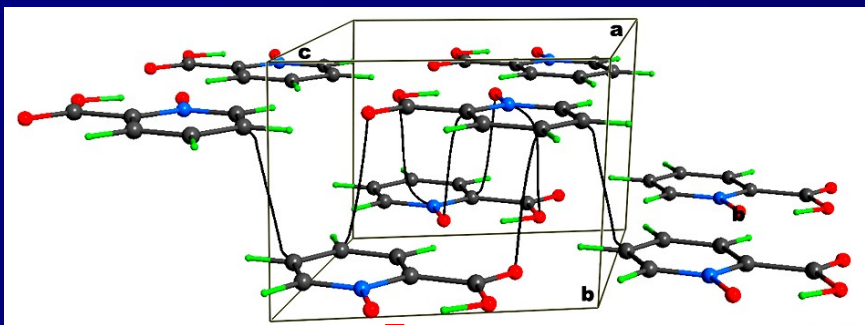
Wyckoff notations and positions of Lagrange points	Type of Lagrange point	
	in $v_{PAEM}(r)$	in $v_{kin}(r)$
16c ($1/8, 1/8, 1/8$) C atom	(3, +3) min	(3, -3) max
16d ($3/8, 3/8, 3/8$) Tetrahedral void	(3, -3) max	(3, +3) min
8a (0, 0, 0) C-C bond	(3, +1)	(3, -1) saddle
8b ($1/2, 1/2, 1/2$)	(3, -1) saddle	(3, +1)

The static force field in NaCl

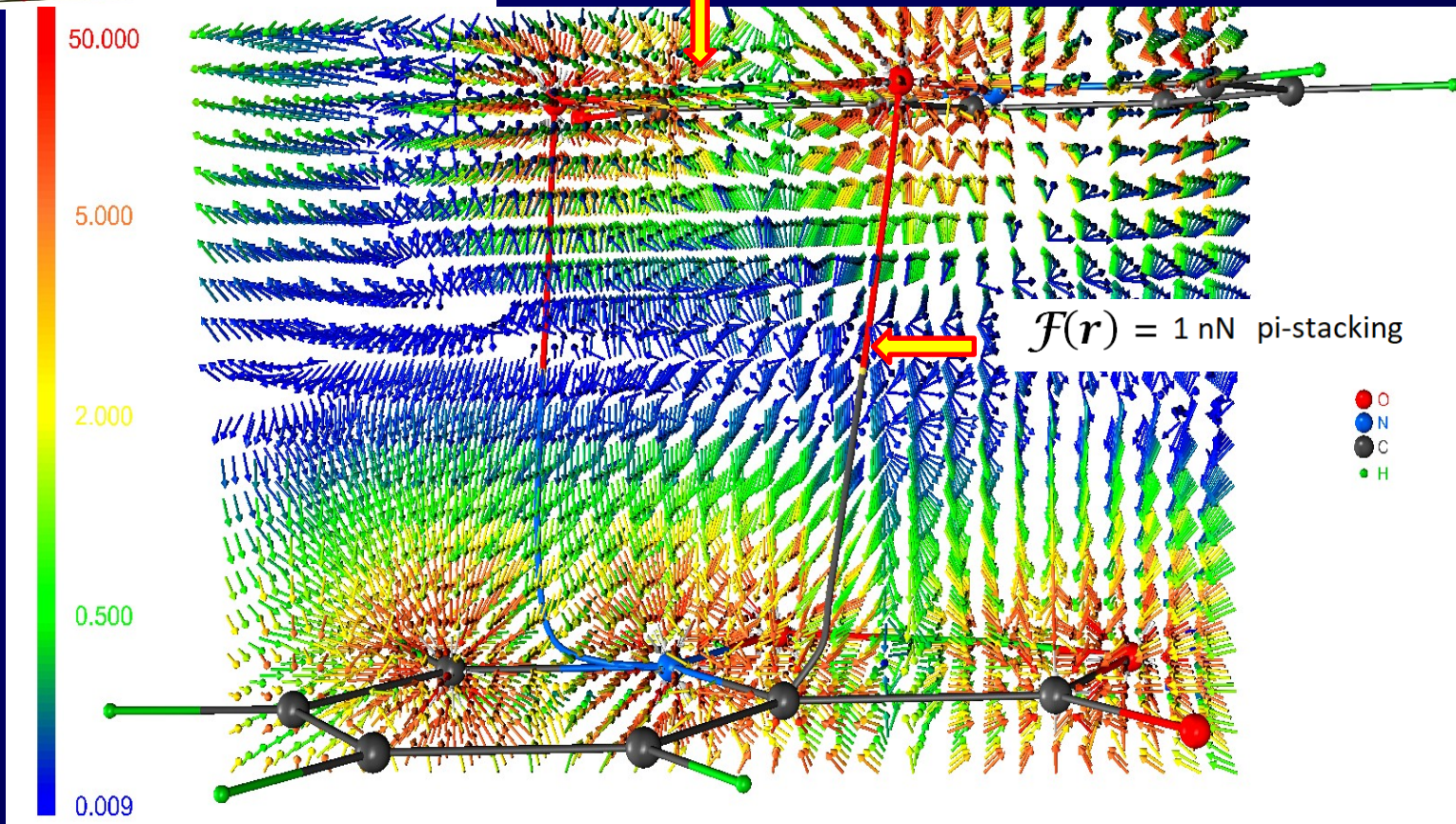


PAEM (3,-3)

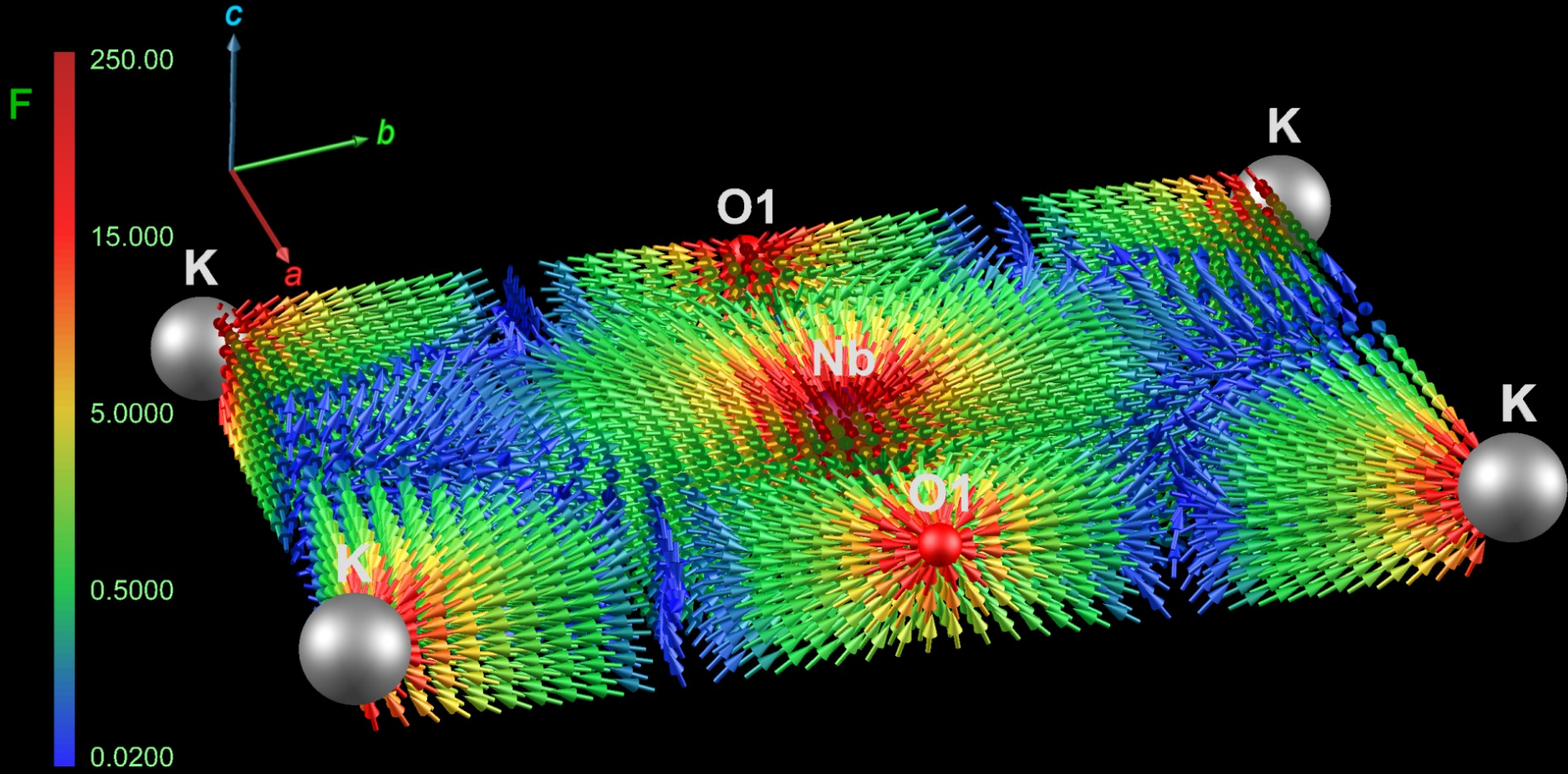
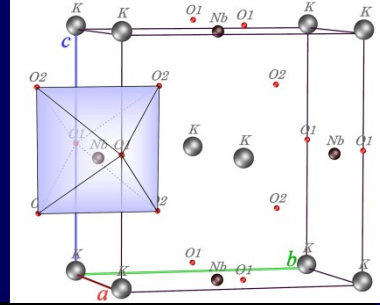
The static force field in crystalline picolinic acid N-oxide, $C_6H_5NO_3$: π -stacking



$\mathcal{F}(r) = 5 \text{ nN}$ covalent bond



The static force field in KNbO_3



Nuclei of neighboring atoms in a crystal are separated in the vector field $\nabla v_{kin}(\mathbf{r})$ by closed surfaces P_i , satisfying the zero-flux condition

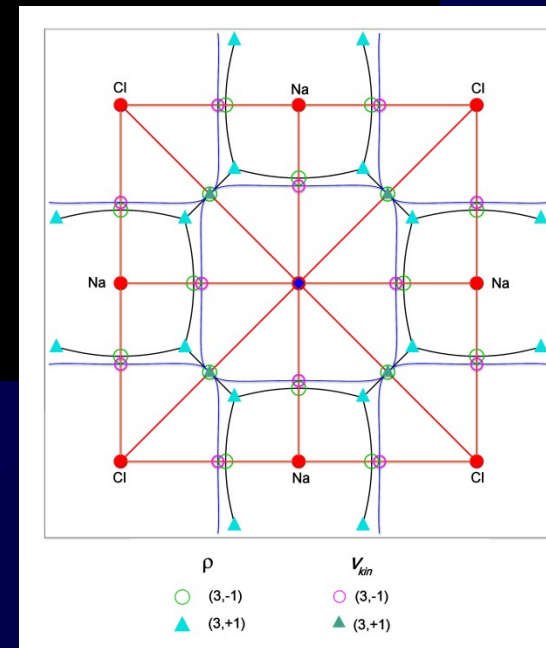
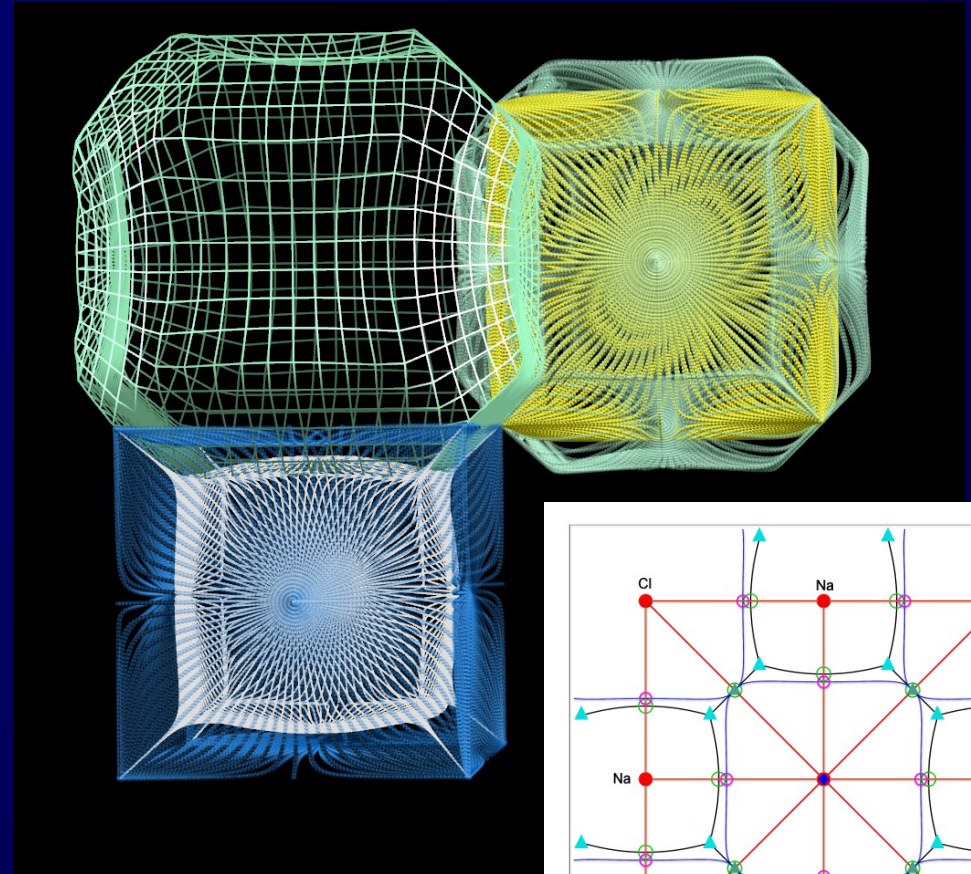
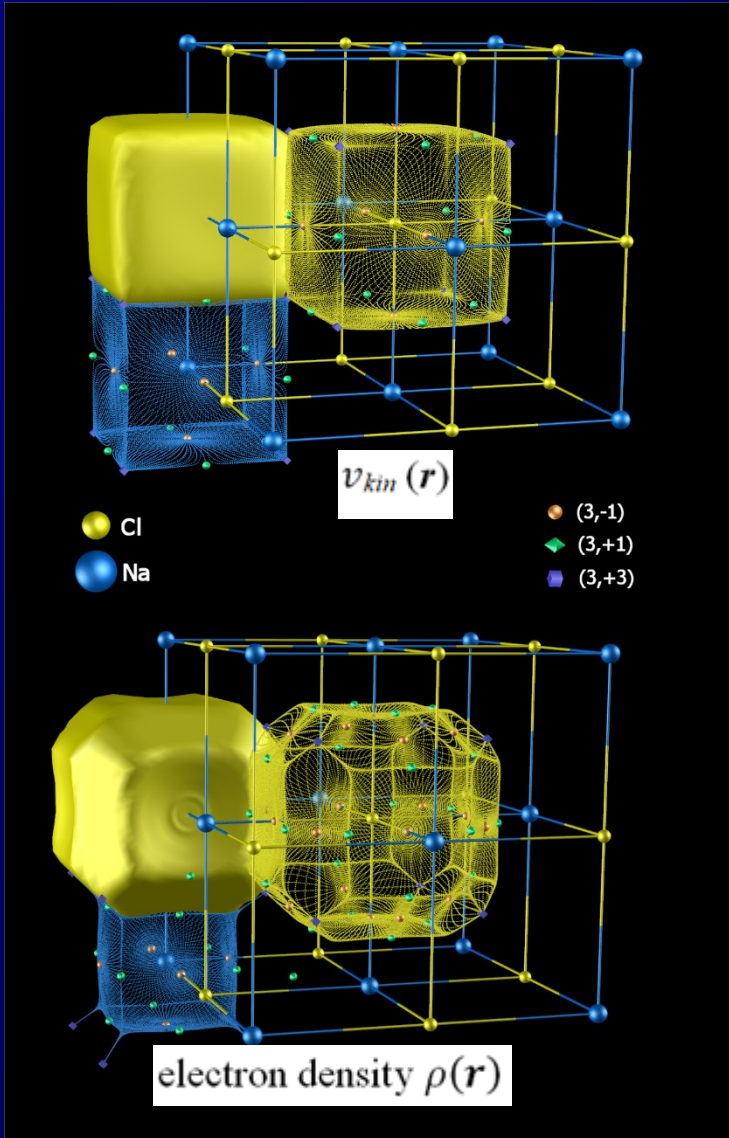
$$\nabla v_{kin}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}) = 0, \quad \forall \mathbf{r} \in P_i(\mathbf{r}),$$

Each volume enveloped by the surface P_i includes a single attractor (atomic nucleus) and defines the v_{kin} -basin of *i-th* bounded atom, *which is in statistical mechanical equilibrium with surrounding* basins.

Atomic-like partition of the force field in a crystal follows from the quantum kinetic properties of the electron continuum (in the field of nuclei). No gradient lines crossing the P-boundary between adjacent equilibrium basins are in the v_{kin} -field. However a unique pair of the gradient lines formed by the two field vectors emitted by the nearest nuclei and ended in the Lagrange point (3, -1) is observed. It mirrors the bond path in electron density linking the bonded atoms in QTAIM

NaCl. The gradient interatomic surfaces
and the Lagrange points

The spatial superposition of the v_{kin} -basins
(Na basin is in blue, Cl basin is in yellow)
and ρ -basins (in white and green, respectively).

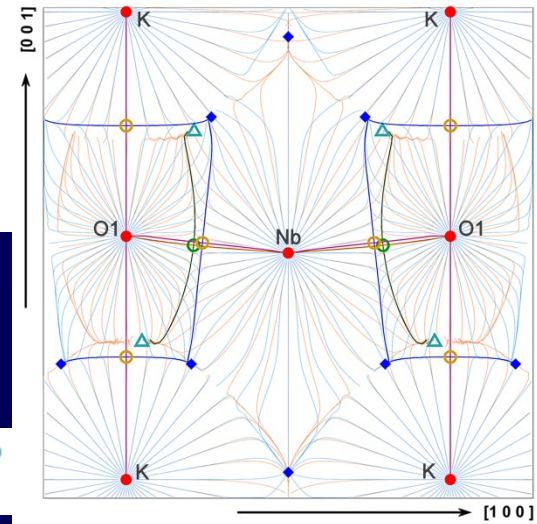
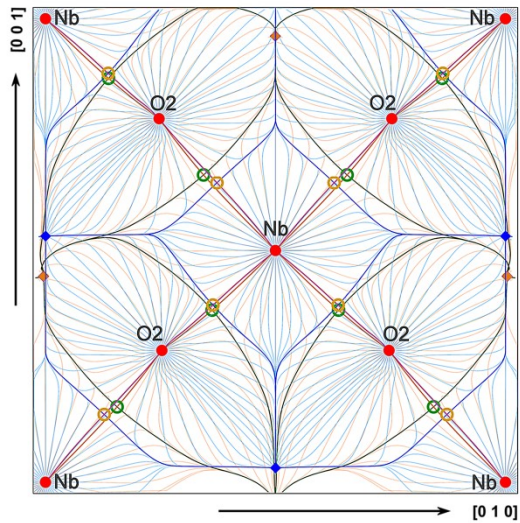


Zero-flux density and force basins in KNbO_3

Superpositions of atomic basins, gradient fields, bond paths in $\rho(\mathbf{r})$ (blue gradient lines) and $v_{\text{kin}}(\mathbf{r})$ (orange lines) and critical points observed in these fields.

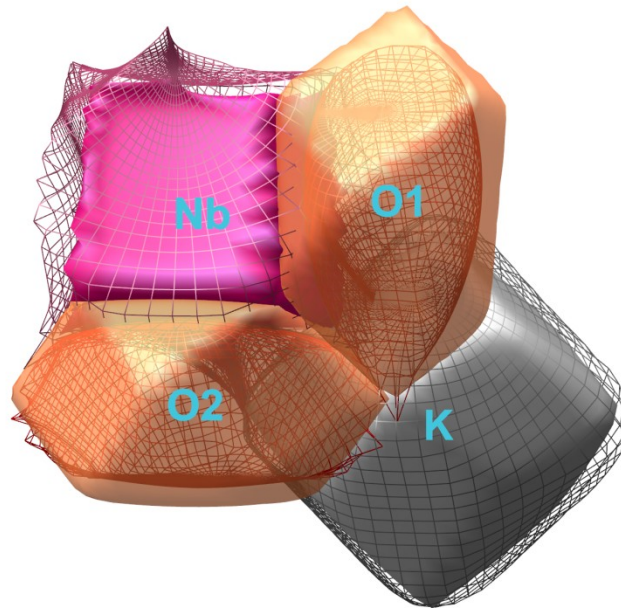
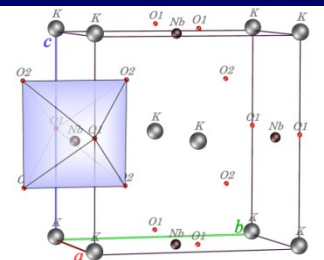
The traces of ED basin boundaries are shown by blue colour, and those for $v_{\text{kin}}(\mathbf{r})$ are of burgundy colour.

KNbO_3 : spatial arrangement of ρ -basins (solid colored surfaces) and v_{kin} basins (mesh surfaces).



(001)

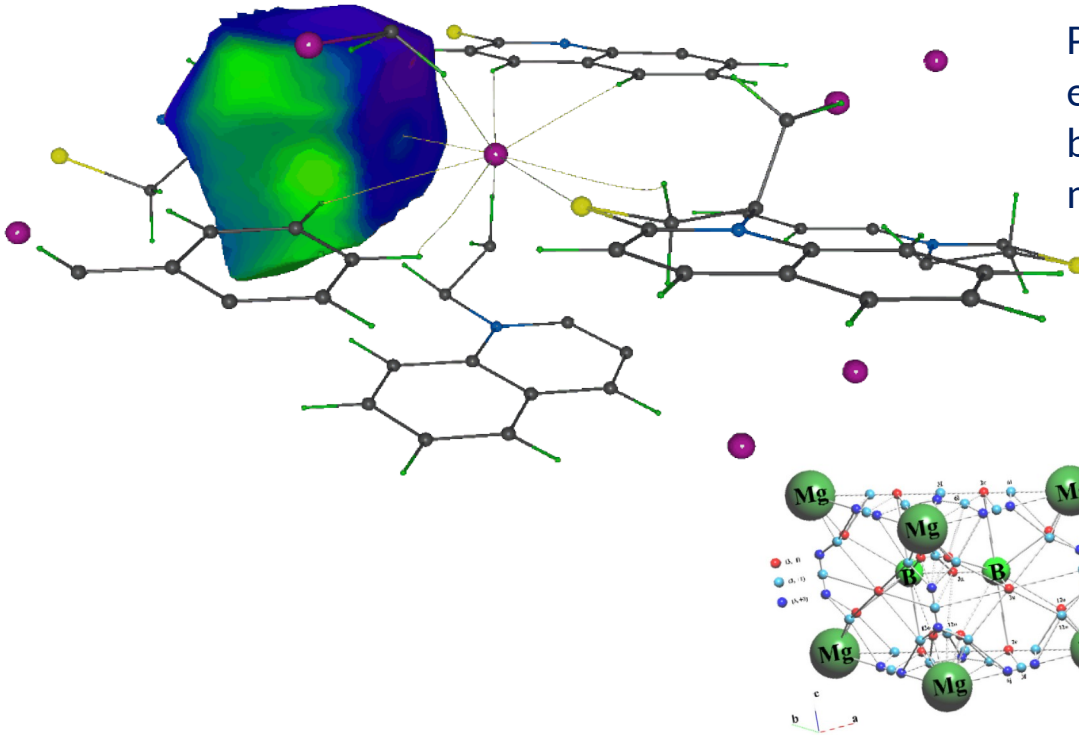
(101)



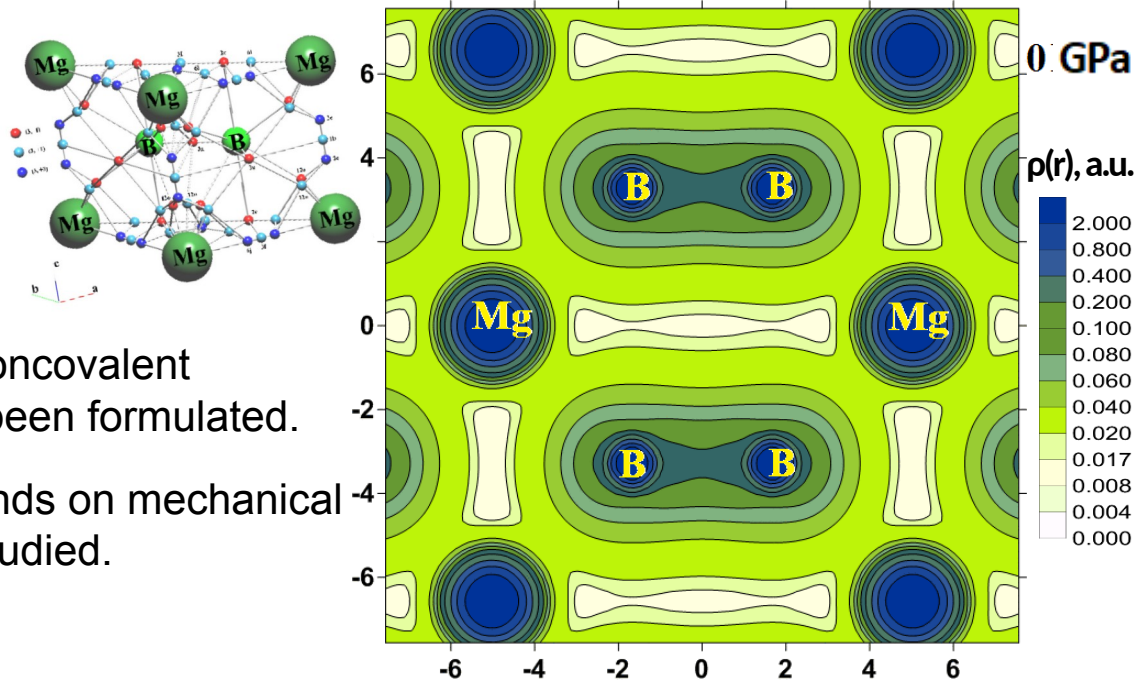
The kinetic forces arising in the v_{kin} – basins of cations are directed towards their surfaces. They penetrate through the surfaces of the electron-density basins of anions and press the first against the second. This ensures the stabilization of the molecular and crystal structure.

Some applications

Potentials mapped on the zero-flux surface of electron density, which determine the atomic basin of Iodine in the thiazinoquinolinium monoiodide crystal



The model illustration of electron density behavior at the hydrostatic pressure in MgB_2 crystal



- ✓ Modern electronic criterion for noncovalent chemical bond categorizing has been formulated.
- ✓ The influence of noncovalent bonds on mechanical properties of crystals has been studied.

Summary

We spread the orbital-free density functional theory to the field of quantum crystallography. The one-electron Euler equation is re-written via electrostatic, exchange, Weizsaecker and Pauli potentials, and the local forces and partial electron densities related with these potentials by the Poisson equation are defined. All these functions were expressed in terms of experimental electron density and its derivatives using the orbital-free DFT approximations, and were applied to the study of inner-crystal features of different crystals.

Our approach introduces the concept of homotropic and heterotropic forces into crystallography and allows decomposing the electron density into physically meaningful components, associated with electrostatics, exchange, and spin-independent wave properties of electrons. As a result, the bonding information about a crystal that was previously unavailable for X-ray diffraction analysis can be now obtained.

Some perspectives for the future:

- *physically-grounded study of the many-faced chemical bonds;*
- *the links between the microscopic and macroscopic properties of materials under pressure and in the external fields;*
- *improving the DFT potentials;*
- *search for realistic pair potentials for atomistic and molecular modeling via AI algorithms;*
- *revealing the improved force field picture;*
- *more...*

Thank you for your attention!